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Density Perturbations in a Brans–Dicke Cosmological Model

Marcelo Samuel Berman[1,2]

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A very general flat solution for Brans–Dicke cosmology with a perfect-fluid, Robertson–Walker metric and a perfect gas law of state is examined regarding density perturbations. The model has growing instabilities, but not of exponential character.

1. INTRODUCTION

The study of density perturbations in cosmology is rewarding, for it offers the best known mechanism for the description of galaxy formation.

Why Brans–Dicke theory? It is well known that BD theoretical framework with a high value for the coupling constant remains as the best alternative to general relativity.

The method adopted here is due to Bandyopadhyay (1977) and was corrected by Baptista *et al.* (1986), who pointed out a small error in the former's calculations.

We shall examine a very general solution for BD theory in a Robertson–Walker flat metric and obeying the perfect gas law of state,

$$p = \rho\alpha \tag{1}$$

where p, ρ, and α are pressure, rest energy density, and a constant. This solution is a particular case of a solution obtained by Morganstern (1971).

2. THE BD FLAT MODEL SOLUTION

It is not difficult to show that the following solution is valid for a BD flat model in the Robertson–Walker metric for a perfect fluid and equation

[1]Department of Astronomy, University of Florida, Gainesville, Florida 32611.
[2]Permanent address: Department of Exact Sciences, Faculdade de Filosofia, Ciencias e Letras da FURJ, 89200 Joinville, SC, Brazil.

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of state (1):

$$R(t)=(mDt)^{1/m} \tag{2}$$

$$\phi(t)=St^{\gamma} \tag{3}$$

$$\gamma=2-\frac{3}{m}(1+\alpha) \tag{4}$$

Here, m and D are nonnull constants, and ϕ is the scalar field defined by

$$\phi(t)=aG^{-1}=\frac{4+2w}{3+2w}G^{-1} \tag{5}$$

Formula (2) represents a constant-deceleration parameter scale-factor model (Berman, 1983; Berman and Gomide, 1988), where the deceleration parameter q is given by

$$q=-\frac{\ddot{R}R}{\dot{R}^2}=m-1 \tag{6}$$

Here, dots stand for time derivatives.

For ρ, we have the following solution:

$$\rho=\rho_0 t^{-3(1+\alpha)/m} \tag{7}$$

$\rho_0=\text{const.}$

3. THE PERTURBATION FORMALISM

The density contrast $u(t)$ and the perturbation for ϕ, to be called $\Psi(t)$, obey following differential system (Baptista *et al.*, 1986):

$$\frac{\ddot{u}}{1+\alpha}+\frac{2\dot{R}\dot{u}}{R(1+\alpha)}-\frac{8\pi\rho u}{\phi}\,\frac{2+w+3(1+w)\alpha}{3+2w}$$
$$=-\frac{8\pi\rho\Psi}{\phi^2}\,\frac{2+w+3(1+w)\alpha}{3+2w}+\frac{2w}{\phi^2}\dot{\phi}\dot{\Psi}-\frac{2w\dot{\phi}^2}{\phi^3}\Psi+\frac{\ddot{\Psi}}{\phi}-\frac{\ddot{\phi}\Psi}{\phi} \tag{8}$$

$$\ddot{\Psi}+\frac{3\dot{R}\dot{\Psi}}{R}-\frac{\dot{u}\dot{\phi}}{1+\alpha}=\frac{8\pi(1-3\alpha)\rho u}{3+2w} \tag{9}$$

4. BEHAVIOR OF DENSITY PERTURBATIONS

It is obvious that a general solution of the above system of equations (8) and (9) is very difficult. But we may show a particular solution that exhibits density instabilities, in a power law of time:

$$\Psi(t)=0 \tag{10}$$

$$u(t)=u_0 t^F \tag{11}$$

Here, u_0 is a constant, and

$$F = -\frac{8\pi(1-3\alpha)(1+\alpha)\rho_0}{(3+2w)\gamma S} \tag{12}$$

subject also to the condition

$$F = \frac{1}{2}\left(\frac{2}{m}-1\right) \pm \frac{1}{2}\left[\left(1-\frac{2}{m}\right)^2 + \frac{32\pi\rho_0(1+\alpha)}{(3+2w)S}\right]^{1/2} \tag{13}$$

For instabilities to exist, we must have $F>0$, and this is probably the most important case, attained, for instance, when

$$w > -1.5, \qquad \alpha < \tfrac{1}{3}, \qquad \gamma < 0$$

i.e.,

$$\frac{1+\alpha}{m} < \frac{2}{3}$$

so that $m<2$.

This is not the unique case. For instance, we can have $F>0$ with $\gamma>0$ if $\alpha>1/3$, is evident from (12).

5. CONCLUSION

We found a growing perturbation for a flat Brans–Dicke (quite general) solution, implying instabilities, though not of an exponential nature, needed to explain galaxy formation. Berman (1990) studied the same problem in general relativity and found a very similar solution.

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